

Statistics

Spring 2023

Lecture 21



Feb 19-8:47 AM

Independent Events:

Two events are independent when one outcome does not change the Prob. of next outcome.

1) Flip a Fair coin

$$P(\text{Tails}) = 0.5, \quad P(\text{Heads}) = 0.5$$

It does not matter what the outcome is, it does not change the prob. of next outcome.

2) Roll a Fair die.

$$P(\text{get } 2) = \frac{1}{6} \text{ on every roll.}$$

3) Multiple-choice questions with 4-choices but only 1 correct Ans

$$P(\text{Correct}) = \frac{1}{4} \text{ on each question.}$$

$$P(\text{Wrong}) = \frac{3}{4}$$

Questions are independent from each other.

4) Draw Cards with replacement

$$P(\text{Same}) = \frac{12}{52} \text{ on every draw.}$$

Mar 14-7:15 AM

IF A and B are independent events, then

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

ex: $P(A) = .5$, $P(B) = .7$, A & B are Independent events

1) $P(\bar{A}) = 1 - P(A) = .5$ 2) $P(\bar{B}) = 1 - P(B) = .3$

3) odds in favor of event A $P(A) : P(\bar{A})$
 $.5 : .5 \rightarrow 1:1$

4) odds against event B
 $P(\bar{B}) : P(B)$
 $.3 : .7 \rightarrow 3:7$

5) $P(A \text{ and } B) = P(A) \cdot P(B) = (.5)(.7) = .35$
Independent events

6) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Addition Rule $= .5 + .7 - .35 = .85$

7) Construct Venn Diagram
 $P(A \text{ only}) = .5 - .35 = .15$
 $P(B \text{ only}) = .7 - .35 = .35$

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Suppose A & B are independent events,
 $P(A) = .3$, $P(B) = .4$.

1) $P(\bar{A}) = .7$ 2) $P(\bar{B}) = .6$

3) odds in favor of event B
 $P(B) : P(\bar{B})$ $.4 : .6 \rightarrow 2:3$

4) odds against event B $\rightarrow 3:2$

5) $P(A \text{ and } B) = P(A) \cdot P(B) = (.3)(.4) = .12$
Ind. Events

6) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Addition Rule $= .3 + .4 - .12 = .58$

7) Make Venn Diagram
 $P(A \text{ only}) = P(A) - P(A \text{ and } B)$
 $= .3 - .12 = .18$
 $P(B \text{ only}) = P(B) - P(A \text{ and } B)$
 $= .4 - .12 = .28$

8) $P(A \text{ only OR } B \text{ only}) = .18 + .28 = .46$

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A box has 2 Red and 8 Blue balls.
 Randomly draw 2 Balls with replacement.

RR RB BR BB
 Sample Space

$P(\text{Red}) = \frac{2}{10}, P(\text{Blue}) = \frac{8}{10}$
 on every draw
 $P(\text{Red}) = .2, P(\text{Blue}) = .8$

$P(\text{RR}) = P(\text{R}) \cdot P(\text{R}) = (.2)(.2) = .04$

$P(\text{RB}) = P(\text{R}) \cdot P(\text{B}) = (.2)(.8) = .16$
 $\rightarrow P(\text{1R \& 1B}) = .32$

$P(\text{BR}) = P(\text{B}) \cdot P(\text{R}) = (.8)(.2) = .16$

$P(\text{BB}) = P(\text{B}) \cdot P(\text{B}) = (.8)(.8) = .64$

# Blue	P(#B)
0	.04
1	.32
2	.64

Blue \rightarrow L1, $P(\text{\#Blue}) \rightarrow$ L2
 use 1-Var Stats with
 L1 & L2
 $\bar{x} = 1.6$ S = Blank $n = 1$
 Total Prob.

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Let's make Tree Diagram

Recall with replacement

$P(\text{RR}) = \frac{2}{10} \cdot \frac{2}{10} = \frac{4}{100} = .04$

$P(\text{RB}) = \frac{2}{10} \cdot \frac{8}{10} = \frac{16}{100} = .16$

$P(\text{BB}) = \frac{8}{10} \cdot \frac{8}{10} = \frac{64}{100} = .64$

$P(\text{BR}) = \frac{8}{10} \cdot \frac{2}{10} = \frac{16}{100} = .16$

$P(\text{1R \& 1B}) = .32$

Suppose we draw 3 balls with replacement.

$P(\text{3 Red Balls}) = P(\text{RRR}) = \frac{2}{10} \cdot \frac{2}{10} \cdot \frac{2}{10} = .008$

$P(\text{3 Blue Ball(s)}) = P(\text{BBB}) = \frac{8}{10} \cdot \frac{8}{10} \cdot \frac{8}{10} = .512$

$P(\text{All Same Color}) = P(\text{All R or All B}) = .008 + .512 = .52$

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R R R .008

Some R
 &.
 Some B

.48

B B B .512

$$P(\text{different color}) = 1 - P(\text{Same color})$$

$$= 1 - .52$$

$$= \boxed{.48}$$

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Class QZ 5

Given $P(A) = .25$

1) $P(\bar{A}) = 1 - P(A) = 1 - .25 = \boxed{.75}$

2) odds in favor of event A. $\rightarrow \boxed{1:3}$
 $P(A) : P(\bar{A}) \quad .25 : .75$

3) odds against event A. $\rightarrow \boxed{3:1}$

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